MAT: Mass Assignment Theory (Basic)

- Mass Assignment (Shafer, Baldwin)
  $m_s : \mathcal{P}(S) \mapsto [\mathit{0}, \mathit{1}]$ such that

$$\sum_{A \subseteq S} m_s(A) = 1$$

- Focal Element A subset $\mathbf{A}$ of a sample space $\mathbf{S}$ such that

$$m_s(A) > 0$$

- Example

$$m(A) = \begin{cases} 0.6 & A = \{a, b\} \\ 0.4 & A = \{b, c\} \\ 0 & \text{otherwise} \end{cases}$$

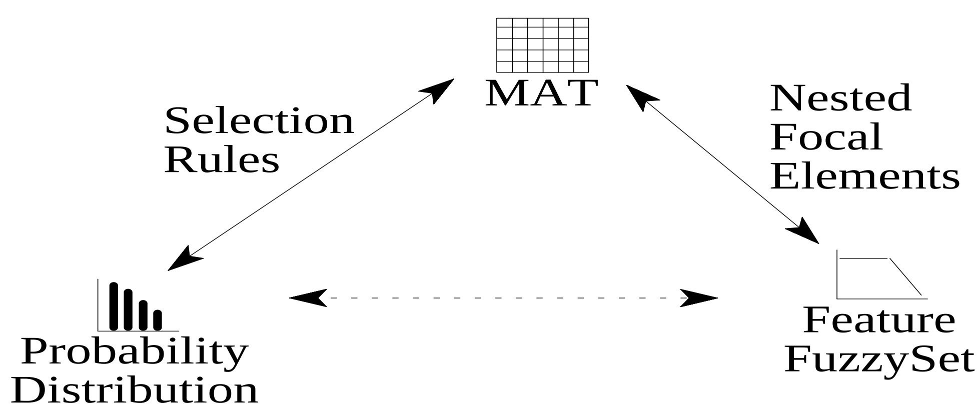
  $\{\mathbf{a}, \mathbf{b}\}$ and $\{\mathbf{b}, \mathbf{c}\}$ are focal elements

- Operations (Baldwin) – Meet, Join, Negation

MAT: Property of Mass Assignment (Glance)

- Q: Probability Distribution $\overset{?}{\Leftrightarrow}$ Fuzzy Set

- A: Mass Assignment + Selection Rules (A. Ralescu)



MAT: Property of Mass Assignment (Eq)

- Mass Assignment and Probability

$$P_s(x) = \sum_{A \subseteq S, x \in A} P_A(x) \cdot m_s(A)$$

- $P_A(x)$: Selection Rules

- Mass Assignment and Fuzzy Set (F)

$$m_F(A) = \begin{cases} \mu_i - \mu_{i+1} & \text{if } A = \{x_1 \ldots xi\} \\ 0 & \text{otherwise} \end{cases}$$

$$1 = \mu_1 \geq \mu_2 \geq \ldots \geq \mu_n > \mu_{n+1} = 0$$

- Probability and Fuzzy Set

$$P_s(x_k) = \sum_{i=k}^{n} P_{A_i}(x_k) \cdot (\mu_i - \mu_{i+1})$$

MAT: Selection Rules

- Selection Rules:
  Probability Distribution on a Focal Element
  Useful for Perception Management

- LPD: Least Prejudged Dist. (Uniform Dist)
  $P_{\{a,b\}}(a) = P_{\{a,b\}}(b) = 0.5$
  $P_{\{b,c\}}(b) = P_{\{b,c\}}(c) = 0.5$

$$\begin{array}{rcl} P_S(a) & = & P_{\{a,b\}}(a) \times m(\{a,b\} \\ & = & 0.5 \times 0.6 = 0.3 \\ P_S(b) & = & P_{\{a,b\}}(b) \times m(\{a,b\}) \\ & + & P_{\{b,c\}}(b) \times m(\{b,c\}) \\ & = & 0.5 \times 0.6 + 0.5 \times 0.4 = 0.5 \\ P_S(c) & = & P_{\{b,c\}}(c) \times m(\{b,c\}) \\ & = & 0.5 \times 0.4 = 0.2 \end{array}$$

- MPD: Most Prejudged Dist. (On One Point)
  $P_{\{a,b\}}(a) = 1.0 \ and \ P_{\{a,b\}}(b) = 0.0$

<u>MAT: Nested Focal Elements</u>

- Example
  $\{a\},\ \{a,b\},\ \{a,b,c\}$

- Fuzzy Set
  $\equiv$
  Mass Assignment w/ <u>Nested</u> Focal Elements

- $\Rightarrow$ Need a Method to Convert Arbitrary MA into MA with Nested Focal Elements

MAT: Tabular Representation

- $sm_{i,j}$: Selection Mass ($P_{A_i}(x_j) \cdot m(A_i)$)
- $m(A_i)$: Mass ($\sum_j sm_{i,j}$)
- $P(x_j)$: Probability ($\sum_i sm_{i,j}$)
- $\sum_{i,j} sm_{i,j} = 1.0$

NFE	x_1	$\cdots$	x_j	$\cdots$	x_n	**MASS**
A_1	$sm_{1,1}$	$-$	$-$	$-$	$-$	$m(A_1)$
$\vdots$	$\vdots$	$\ddots$	$-$	$-$	$-$	$\vdots$
A_i	$sm_{i,1}$	$\cdots$	$sm_{i,j}$	$-$	$-$	$m(A_i)$
$\vdots$	$\vdots$	$\cdots$	$\cdots$	$\ddots$	$-$	$\vdots$
A_n	$sm_{n,1}$	$\cdots$	$sm_{n,j}$	$\cdots$	$sm_{n,n}$	$m(A_n)$
PROB	$P(x_1)$	$\cdots$	$P(x_j)$	$\cdots$	$P(x_n)$	1.0

MAT: Tabular Representation (Example)

$$m(A_1 : \{x_1\}) = 0.1$$
$$m(A_2 : \{x_1, x_2\}) = 0.2$$
$$m(A_3 : \{x_1, x_2, x_3\}) = 0.3$$
$$m(A_4 : \{x_1, x_2, x_3, x_4\}) = 0.4$$

$$P_{A_i}(x) = 1/i \; \forall \; i = 1, 2, 3, 4$$

NFE	x_1	x_2	x_3	x_4	$m(A)$	$\mu(x)$
A_1	0.1	-	-	-	0.1	1.0
A_2	0.1	0.1	-	-	0.2	0.9
A_3	0.1	0.1	0.1	-	0.3	0.7
A_4	0.1	0.1	0.1	0.1	0.4	0.4
$P(x)$	0.4	0.3	0.2	0.1	1.0	n/a